WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2022-23

PHSACOR05T-PHYSICS (CC5)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any *ten* questions from the following: $2\times10=20$

(a) Let $F(x)$ have a Fourier Series expansion

$$F(x)=\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

then prove that, $\langle F^2(x)\rangle=\frac{1}{2\pi}\int_{-\pi}^{\pi}F^2(x)dx=\sum_{n=1}^{\infty}\left(\frac{a_n^2+b_n^2}{2}\right)$

(b) Can $y=\tan x$ be expanded in a Fourier Series? Explain.

(c) Verify whether $y_1(x)=\sin\sqrt{x}$ and $y_2(x)=\cos\sqrt{x}$ are linearly independent or not.

(d) From the generating function $G(z,h)=(1-2zh+h^2)^{-1/2}=\sum_{n=0}^{\infty}P_n(z)h^n$, determine $P_3(z)$.

(e) Prove that $J_{n+1}(x)+J_{n-1}(x)=\frac{2n}{x}J_n(x)$.

(f) Prove that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.

(g) Express $f(x)=6x^2+7x+2$ in terms of Legendre polynomials.

(h) Write down the orthogonality properties of Hermite polynomial.

(i) Evaluate $\Gamma(\frac{5}{2})$ using $\Gamma(\frac{1}{2})=\sqrt{\pi}$.

(j) Lagrangian of a point mass (m) under gravity (g) is given by

$$L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)-mgh$$

What are the cyclic coordinates for the system?

(k) Show that the general solution of the wave equation

$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ is of the form $y=f_1(x-ct)+f_2(x+ct)$.

(l) Find the Hamiltonian for a particle moving in a rotating frame.

(m) State Hamilton's principle.

(n) Prove that

(i) $[F,G]=-[G,F]$ and

(ii) $[cF,G]=c[F,G]$ where c = constant. and [ ] = Poisson bracket.

2. (a) Expand as a Fourier Series 4

$$f(x)=x^2+x \text{ for } -\pi \le x \le \pi$$

(b) Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ and find the value of $\beta\left(\frac{3}{2},\frac{1}{2}\right)$. 3+1

(c) Show that $P_n(-x)=(-1)^n P_n(x)$ 2

3. (a) Using the generating function for the Hermite polynomial $H_n(x)$ expressed as 2+2

$$e^{2tx-t^2}=\sum_{n=0}^{\infty}\frac{1}{n!}t^n H_n(x)$$

Solve the following recurrence relation

(i) $2nH_{n-1}(x)=H_n'(x)$

(ii) $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$

(b) For the Legendre polynomials, show that 3

$$P_{2n}(0)=(-1)^n\frac{(2n-1)!!}{(2n)!!}$$

(c) Consider an electric charge q placed on the z-axis at $z=a$. Show that the electric potential at a non-axial point having position vector $\vec{r}$ is given by 3

$$V=\frac{q}{4\pi\varepsilon_0 r}\sum_{n=0}^{\infty}P_n(\cos\theta)\left(\frac{a}{r}\right)^n$$

Where $P_n(\cos\theta)$ are Legendre Polynomials.

4. (a) Using Hamilton's Canonical equations, derive the equation of motion of a particle moving in a force field in which the potential is given by $V=-\dfrac{k}{r}$, where k is positive constant. 4

(b) Given the Lagrangian $L=\dfrac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-V(r)$. Find the Hamiltonian and hence the equations of motion. 3

(c) Prove that $(n+1)P_{n+1}=(2n+1)xP_n-nP_{n-1}$. 3

5. (a) Solve the differential equation 3

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2} \text{ if } u(x,0)=\sin\pi x.$$

(b) An electric dipole with opposite charges of masses m_1 and m_2 separated by a distance l is placed in an external homogeneous electric field. Write down the Lagrangian of the dipole. 3

(c) Apply Legendre Transformation on the Internal energy function $U=U(S,V)$ to obtain Helmholtz free energy $F=F(T,V)$. 2

(d) If ψ is a solution of Laplace's equation, show that $\dfrac{\partial\psi}{\partial z}$ is also a solution. 2

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